Roll No. _____

Code : 11201819M-A

Class : XI
MATHEMATICS

Time Allowed : 3 Hours   Maximum Marks : 100

General Instructions :

1. Question paper consists of 29 questions dividied into four sections A, B, C and D.
   Section A consists of 4 questions of 1 mark each.
   Section B consists of 8 questions of 2 marks each.
   Section C consists of 11 questions of 4 marks each.
   Section D consists of 6 questions of 6 marks each.
2. There is no overall choice. However, internal choices are provided in some questions. In these questions, you have to attempt one out of the given options.
3. Use of calculator is not allowed.

SECTION - A

1. If $\sin x = \dfrac{3}{5}$ where $0 < x < \dfrac{\pi}{4}$, then find $\cos 2x$.   1

2. Write the conjugate of complex number $(1 + i)^2$.   1

3. Find the equation of circle whose centre is at $(0, 0)$ and of radius 5 cm.   1

4. Write the converse of the following statement :   1

   "If a number is even, then x^2 is also even."

OR

   Write the contrapositive of the following statement

   "If you are born in India, then you are a citizen of India"

SECTION - B

5. Given that $A = \{2x : x \in N, 1 \le x < 4\}$ and   2

   $B = \{x+2 : x \in N, 2 \le x < 6\}$

   Find $B - A$

6. Using properties of sets, prove that for all sets A and B   2

   $(A \cap B) \cup (A - B) = A$

OR

   Show that $A \cup B = A \cap B$ implies $A = B$.

7. Let A = {1, 3, 5}. Define a relation R from A to A by
   R = {(x, y) : x + y > 6; x, y ∈ A}
   (i) Write R in roster form.
   (ii) Write its domain and range.   2

8. Find the value of n if $^nP_5 = 20\,^nP_3$.   2

9. The centroid of a triangle ABC is at the point (0, 1, 2). If coordinates of A and B
   are (2, –3, 6) and (0, 8, –5) respectively. Find the coordinates of the point C.   2

OR

Show that the points P(–2, 3, 5), Q(1, 2, 3) and R(7, 0, –1) are collinear.

10. Find the derivative of $f(x) = \dfrac{x+2}{x}$, $x \neq 0$. w.r.t.x.   2

OR

Find the derivative of $g(x) = x \cos x$ w.r.t.x.

11. Identify the quantifier in the following statement and write the negation of the
    statement.   2
    "There exist a number which is equal to its square."

12. A sports teacher wants to select a team of 3 chess players from 2 girls & 3 boys.
    What is the probability that team will have 1 girl and 2 boys?   2

SECTION - C

13. In a group of 50 students, 14 drink orange juice but not eat apple, 30 drink orange
    juice and each student like at least one of them. Find   4
    (i) How many drink orange juice as well as eat apple?
    (ii) How many eat apple but not drink orange juice?

14. Find the domain and range of the real function   4

    $$f(x) = \frac{1}{4 - x^2}$$

15. Find the general solution of the equation   4
    $\cos x + \cos 2x + \cos 3x = 0$.

16. Prove that : $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$.   4

17. If $(x - iy)^{\frac{1}{3}} = a - ib$ where $x, y, a, b \in$ R, show that

$$\frac{x}{a} - \frac{y}{b} = -2\left(a^2 + b^2\right)$$

4

18. A student wants to make words with or without meaning by using all letters of the word "ORIGIN"

    (i)   How many maximum number of words, student can make?

    (ii) If these words are written as in dictionary. What will be the 181th word?

4

19. In the expansion of $(1 + x^2)^8$, find the difference between the coefficients of x^6 and x^4.

4

OR

If x is a real number and its middle term in the expansion of $\left(\frac{x}{3} - 3\right)^8$ is 1120. Find x.

20. The sum of two positive numbers is 4 times their geometric mean, show that numbers are in the ratio $(2 + \sqrt{3}) : (2 - \sqrt{3})$.

4

21. An arch is in the form of a semi-ellipse. It is $8m$ wide and $2m$ high at centre. Find the height of the arch at a point $1m$ from one end.

4

OR

Find the equation of the hyperbola whose foci are at $(0, \pm 3)$ and length of latus rectum is 16.

22. Evaluate : $\lim\limits_{x \to 0} f(x)$ where $f(x) = \begin{cases} 3x+2, & x \le 0 \\ 2(x+1), & x > 0 \end{cases}$

4

OR

Find the derivative of $\cos 2x$ w.r.t. x using first principle.

23. Two candidates Sunil and Ravi appeared in an interview. The probability that Sunil will qualify the interview is 0.04 and that Ravi will qualify the interview is 0.2. The probability that both will qualify the interview is 0.03. Find the probability that

    (i)   Both Sunil and Ravi will not qualify the interview.

    (ii) Only one of them will qualify the interview.

4

SECTION - D

24. Prove that : $\sin 3x \cos^3 x + \cos 3x \sin^3 x = \dfrac{3}{4} \sin 4x$ 6

OR

Prove that :
$$\sin^2\alpha + \sin^2(\alpha - \beta) - 2\sin\alpha.\cos\beta.\sin(\alpha - \beta) = \sin^2\beta.$$

25. Using principle of mathematical induction, prove that   6

$$3.2^2 + 3^2.2^3 + 3^3.2^4 + \ldots\ldots\ldots +3^n.2^{n+1} = \dfrac{12\left(6^n - 1\right)}{5},$$

for all $n \in N$.

OR

Using principle of matematical induction, prove that $41^n - 14^n$ is a multiple of 27 for all $n \in N$.

26. Find the solution region for the following inequalities :   6

$$2x + y \geq 2$$
$$y - x \geq -1$$
$$x + 2y \leq 8$$
$$x, y \geq 0$$

Also find the coordinates of the vertices of the solution region.

27. Prove that :   6

$$\dfrac{3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \ldots\ldots\ldots + (2n+1) \times n^2}{1 + (1+2) + (1+2+3) + \ldots\ldots\ldots + (1+2+3+\ldots\ldots + n)} = \dfrac{3n^2 + 5n + 1}{n+2}$$

28. A line is such that its segment between the lines $4x + 3y - 21 = 0$ and $10x + y - 59 = 0$ is bisected at the point (4, 6). Find its equation.   6

OR

The image of a point with respect to the line $2x - y + 6 = 0$, assuming the line to be a mirror, is (6, 6). Find the point. Also find the equation of line joining this point and its image.

29. Find the mean deviation about mean for the following data :   6

Class	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	4	5	12	7	13	6	3